Small-scale Fading

Radio Channel

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Overview

Small-scale fading

Definition: Rapid fluctuations of a radio signal's amplitude, phase, or multipath delay over a short time or distance.

Causes: Multiple versions of the transmitted signal (multipath waves).The main factors include multipath propagation, Doppler effect (TX-RX motion), surrounding objects' movement, and the transmitted signal bandwidth.

Effect:

- Constructive / destructive interference -> Deep fading
- Random frequency modulation
- Time dispersion (echoes)

Doppler Effect

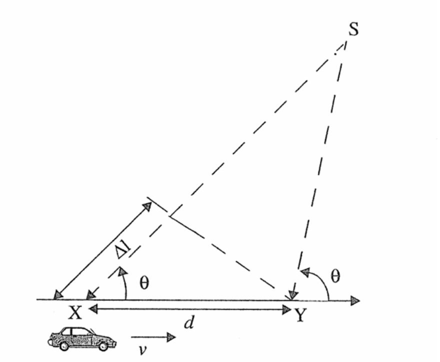
Doppler shift of a mobile signal is given by:

$$f_d = \frac{v}{\lambda} \cos \theta$$

where λ is the wavelength of the signal from S to mobile.

Explanation:

- Received frequency: $f_r = f_c + f_d, \; f_c = \frac{c}{\lambda}$
- Moving toward source → frequency increases
- Moving away from source → frequency decreases
- Perpendicular motion → no Doppler shift
- In multipath: different paths have different Doppler shifts → Doppler spreading (wider bandwidth)



Impulse Response Model

A mobile radio channel can be modeled as a *Linear Time-Varying (LTV) filter*, that is, a linear filter with a time varying impulse response.

General model:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t,\tau) d\tau$$

- t : time variations due to motion
- τ : channel multipath delay for a fixed value of t
- $x(t)$: transmitted signal
- $y(t)$: received signal
- $h(t,\tau)$: the impulse response of the time varying multipath radio channel

Baseband Equivalent Model

A bandpass signal can be expressed as a baseband signal (complex envelope) multiplied by a carrier. Bandpass signals:

$$x(t) = \Re\{c(t)e^{j2\pi f_c t}\}, \qquad y(t) = \Re\{r(t)e^{j2\pi f_c t}\}$$

After removing the high-frequency carrier, we only study the baseband signals.

Baseband equivalent model:

$$r(t) = \tfrac{1}{2}\int_{-\infty}^{\infty} c(\tau)\, h_b(t,\tau)\, d\tau$$

- $c(t)$: baseband transmitted signal
- $r(t)$: baseband received signal
- $h_b(t,\tau)$: baseband channel impulse response

Time Delay

In a multipath channel, signals from different propagation paths arrive at different delays. Therefore, the delay variable τ is continuous, which means there can be infinitely many possible paths.

By discretizing the continuous delay axis τ into bins of width $\Delta\tau$ (called **excess delay bins** axis), all signals arriving within the same bin are grouped together and treated as one effective path.

Definitions

- $\tau_0 = 0$: delay of the first arriving signal (chosen as reference)
- τ_i: excess delay of the i-th path, relative to τ_0
- $\Delta\tau$: delay bin width (time resolution)
- N: total number of delay bins

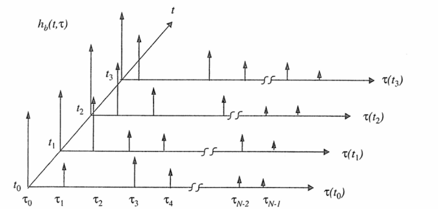
Discrete Baseband Impulse Response

Based on the concept of delay bins, a multipath radio channel can be modeled as the superposition of delayed and scaled impulses.

Discrete baseband impulse response:

$$h_b(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau) e^{j\theta_i(t,\tau)} \delta(\tau - \tau_i(t))$$

- $a_i(t,\tau)$: attenuation of path i
- $\theta_i(t,\tau)$: phase shift of path i
- $\tau_i(t)$: propagation delay of path i
- $\delta(\tau - \tau_i)$: impulse at delay τ_i



Power Delay Profile (PDP)

Since the delay axis is discretized into bins, the **power delay profile (PDP)** describes the average received power at different excess delays.

Definition:

$$P(\tau) = \mathbb{E}\left[|h_b(t,\tau)|^2\right]$$

To approximate the ensemble average in practice, we fix the delay τ and take a spatial average of $|h_b(t,\tau)|^2$ over a local area.

Normalize to PMF:

$$p(\tau_k) = \frac{P(\tau_k)}{\sum_j P(\tau_j)}, \quad \sum_k p(\tau_k) = 1$$

After normalization, $p(\tau_k)$ can be viewed as the distribution of power across delay bins, enabling the computation of statistical moments.

Time Parameters

From the PMF $p(\tau_k)$ of PDP, we can define the following:

Mean Excess Delay:

$$\bar{\tau} = \sum_k \tau_k \, p(\tau_k)$$

The average arrival time of multipath power.

Second Moment of Delay:

$$\overline{\tau^2} = \sum_k \tau_k^2 \, p(\tau_k)$$

Measures average squared delay.

RMS Delay Spread:

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

Indicates how widely multipath power spreads around the mean delay.

Threshold

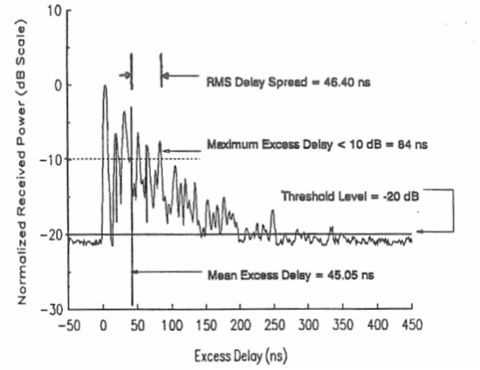
- **Threshold level**:
  A cutoff used to separate multipath components from noise.
- **Maximum excess delay (at X dB)**:
  Let $P_{\max}$ be the maximum PDP power. Define

  $$\tau_X = \max\{\tau : P(\tau) \geq P_{\max} \cdot 10^{-X/10}\}$$

  Equivalently,

  $$P(\tau_X) \geq P_{\max} - X \text{ dB}.$$

  It quantifies the latest significant multipath component within X dB of the strongest path.



Coherence Bandwidth

PDP and the magnitude frequency response $H(f)$ of a mobile radio channel are related through the Fourier transform. Therefore, time parameters have their duality in frequency domain.

Coherence Bandwidth B_c:
A frequency range over which the channel passes all spectral components with approximately equal gain and linear phase. In other words, within B_c, the channel can be considered *flat*.

Dual parameters:

$$B_c \approx \frac{1}{50\sigma_\tau} \quad (\rho > 0.9), \qquad B_c \approx \frac{1}{5\sigma_\tau} \quad (\rho > 0.5)$$

where $\rho =$ correlation between $H(f)$ and $H(f + \Delta f)$.

Doppler Spread and Coherence Time

Doppler Spread B_D:
When a Doppler shift f_d on a signal with carrier frequency f_c is transmitted, the received signal spectrum is in the range:

$$[f_c - f_d\ ,\ f_c + f_d]$$

Coherence Time T_c:
Time duration over which the channel can be considered time-invariant. Coherence time is the dual parameter of doppler spread:

$$T_c \approx \frac{1}{f_m}$$

where $f_m = v/\lambda$ is the maximum Doppler shift.

Types of Small-Scale Fading

The type of fading depends on parameters:

- **Signal parameters**:
  Bandwidth B_S (dual is symbol period T_S)
- **Channel parameters**:
  Coherence bandwidth B_C (dual is rms delay spread σ_τ), coherence time T_C (dual is Doppler spread B_D)

Two fading effects:

- **Multipath time delay spread**:
  Flat fading and frequency selective fading
- **Doppler spread**:
  Fast fading and slow fading

The two fading effects are independent, leading to four possible types of fading (Flat/Freq-Selective × Fast/Slow).

Flat Fading

Definition: If the channel gain and phase are nearly constant over the signal bandwidth, the channel induces only amplitude variations while preserving the spectral shape.

Frequency domain view:

- Narrowband (signal) condition: $B_S \ll B_C$
- Channel appears flat in frequency

Time domain view:

- Equivalent condition: $T_S \gg \sigma_\tau$
- Multipath delays are much smaller than the symbol period
- Gain fluctuates over time

Model: Narrowband channel, amplitude varying channel and Rayleigh fading model.

Frequency Selective Fading

Definition: If the channel gain and phase are not constant over the signal bandwidth, the received signal experiences distortion due to multipath delay spread.

Frequency domain view:

- Wideband (signal) condition: $B_S > B_C$
- Channel gain varies with frequency

Time domain view:

- Equivalent condition: $T_S < \sigma_\tau$
- Multipath delay spread is larger than symbol period
- Causes **intersymbol interference (ISI)**

Model: Wideband channel and two-ray Rayleigh fading model.

Fast Fading

Definition: Channel impulse response changes rapidly within one symbol duration. The coherence time is smaller than the symbol period, leading to time-selective distortion.

Time domain view:

- Condition: $T_S > T_C$
- Channel varies faster than the symbol duration
- Causes rapid time-selective distortion

Frequency domain view:

- Equivalent condition: $B_S < B_D$
- Doppler spread dominates relative to signal bandwidth

Slow Fading

Definition: Channel impulse response changes much more slowly than the transmitted baseband signal. The channel can be assumed static over several symbol durations.

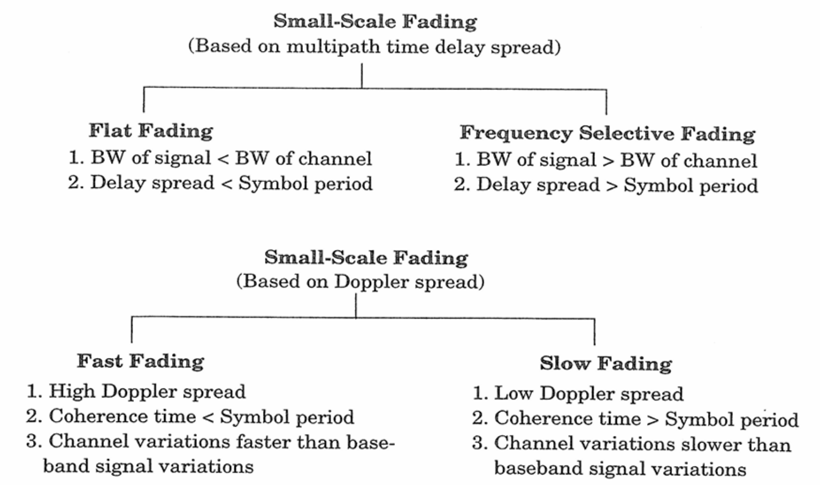
Time domain view:

- Condition: $T_S \ll T_C$
- Channel remains nearly constant during a symbol
- Signal experiences slow variations in amplitude and phase

Frequency domain view:

- Equivalent condition: $B_S \gg B_D$
- Doppler spread is much smaller than signal bandwidth

Fading Effects



Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading

1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading

1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

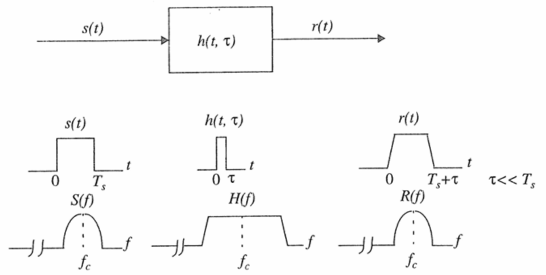
Fast Fading

1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

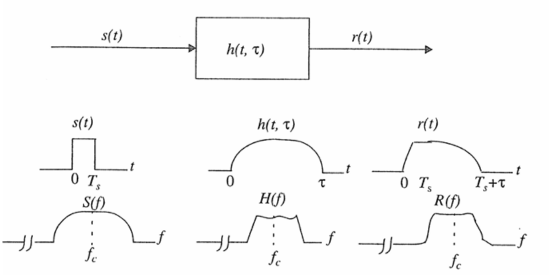
Slow Fading

1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Flat v.s. Frequency Selective Fading

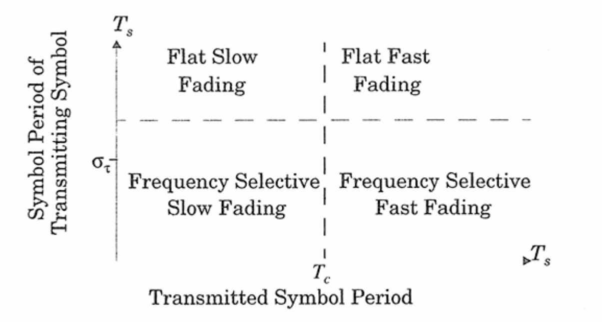


Flat Fading

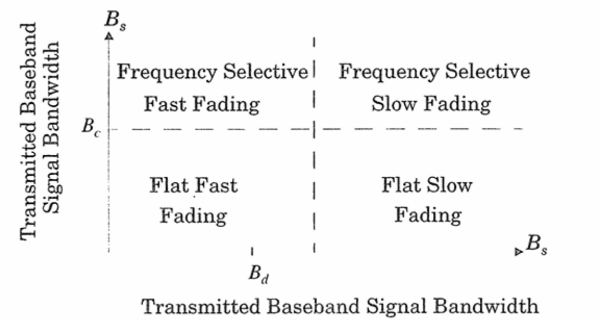


Frequency Selective Fading

Flat/Freq-Selective × Fast/Slow Fading



Based on T_s vs. σ_τ, T_c



Based on B_s vs. B_c, B_D

Rayleigh Fading Distribution

Definition: Describes the envelope distribution of a received signal in *non-LOS channels* (no dominant path). The received signal is the sum of many scattered multipath components.

PDF:

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp\left(-\dfrac{r^2}{2\sigma^2}\right), & r \geq 0 \\ 0, & r < 0 \end{cases}$$

Insight:

- No LOS component $\Rightarrow$ envelope $\sim$ Rayleigh
- Small r values are frequent; strong signals are rare.

Rayleigh Fading Channel Model

Standard Rayleigh fading only gives the envelope distribution at one time instant, but to capture delay structure (different paths with different τ_i), we need an multipath model.

Single-path model

$$h_b(t) = \alpha_1 e^{j\phi_1} \delta(t)$$

Multi-path model

$$h_b(t) = \sum_{i=1}^{N} \alpha_i e^{j\phi_i} \delta(t - \tau_i)$$

Where:

- α_i: independent Rayleigh-distributed amplitude
- ϕ_i: independent random phase $\sim U[0, 2\pi]$
- τ_i: propagation delay of path i

Rician Fading Distribution

Definition: Describes the envelope distribution when a *dominant LOS component* exists along with multiple scattered multipaths.

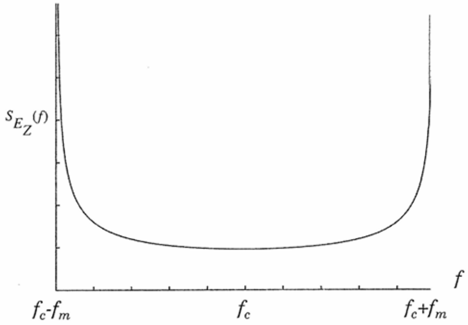
PDF:

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right), & r \geq 0 \\ 0, & r < 0 \end{cases}$$

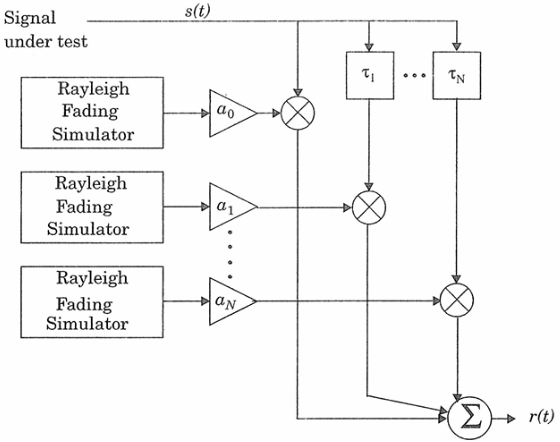
Insight:

- LOS $+$ multipath $\Rightarrow$ envelope $\sim$ Rician
- Characterized by the *Rician K-factor*: $K(dB) = 10 \log \frac{A^2}{2\sigma^2}$
- As $A \to 0$ ($K \to -\infty$ dB), Rician $\to$ Rayleigh.

Appendix



Doppler spread
(Y-axis is power spectrum density)



Multi-path model
(Simulate frequency selective channel)

The End