Radio Channel

Large-scale Fading

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Overview

Background

Wired channel: Stationary and predictable.

Radio channel:
Transmission path (i.e., between transmitter and receiver, TX/RX) can vary from simple line-of-sight to severely obstructed paths.
Mobility (TX/RX movement) introduces fading.

$\rightarrow$ *Fundamental limitations on system performance.*

Radio Channel Modeling

Radio channels are highly random and difficult to analyze.
$\rightarrow$ *Empirical measurements in real environments are required.*
$\rightarrow$ *Models are typically statistical, based on measured data.*

Fading

Radio wave propagation occurs through electromagnetic waves, including reflection, diffraction, and scattering.

Most radio systems operate in urban environments, where multiple propagation paths exist between TX/RX.

$\rightarrow$ *The interaction of these waves causes multipath fading.*
$\rightarrow$ *At certain locations, signal strength may increase or decrease significantly.*

Another type of fading is due to path loss:
As the T-R separation distance (distance between TX/RX) increases, the received signal strength decreases.

Large-scale Fading

Path loss (due to large T-R separation distances) and
shadowing (e.g., caused by buildings obstructing the signal path)
are collectively referred to as large-scale fading.

$\rightarrow$ *Characterized by the average and variability of the received signal strength over large distances.*

Large-scale Propagation Model

Predicts the mean and variance of received signal power over large T-R distances (typically hundreds or thousands of meters).

Small-scale Fading

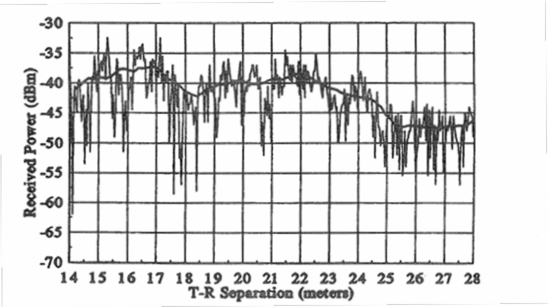
Multipath propagation and
Doppler effects (caused by movement of TX/RX or reflectors)
cause rapid fluctuations in the received signal strength, known as small-scale fading.

→ *Since this is a random process, statistical models are used to describe it.*

Small-scale Fading Model

Characterizes small-scale fading using distributions such as Rayleigh or Rician.
Applicable over distances of a few wavelengths or over short time intervals (on the order of seconds).

Summary: Large-scale vs. Small-scale Fading



Feature	Large-scale Fading	Small-scale Fading
Cause	Path loss, shadowing	Multipath, Doppler
Scale	100s—1000s of meters	Few wavelengths / seconds
Effect	Gradual signal strength change	Rapid fluctuations
Model	Propagation models (mean, variance)	Statistical models (Rayleigh, Rician)
Use	Coverage prediction	Channel behavior modeling

Free Space Propagation Model

Scenario: Line-of-sight (LOS) with no obstruction between TX/RX.

Used in:

- Satellite communications
- Microwave LOS radio links

Key Idea:

- Received power decreases proportionally to $\frac{1}{d^2}$
- Often used as a baseline to define reference power at distance d_0

Limitations:

- Only valid in the far-field
- Ignores multipath, reflection, shadowing

Friis Free Space Equation

Friis Free Space Equation:

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$

- $P_r(d)$: Received power at distance d from TX
- P_t: Transmitted power
- G_t, G_r: Antenna gains
- $\lambda = c/f$: Wavelength
- $L \geq 1$: System loss factor (non-propagation losses)

Path Loss (in dB):

$$PL(d)\,[\text{dB}] = 10\log\left(\frac{P_t}{P_r}\right) = 10\log\left(\frac{(4\pi)^2 d^2}{G_t G_r \lambda^2}\right)$$

Path loss is the difference (in dB) between the effective transmitted and received power.

Far-Field

Far-field distance d_f:

$$d_f = \frac{2D^2}{\lambda} \quad \text{where } D = \text{largest dimension of antenna aperture}$$

Valid when:

$$d_f \gg D \quad \text{and} \quad d_f \gg \lambda$$

Electromagnetic field regions:

Region	**Distance range**	**Field behavior**
Near field	$\ll \lambda$	No clear wave propagation
Fresnel (transition)	$\sim$ several λ	Wavefronts are bending, not fully planar
Far field	$\gg \lambda$, $d > d_f$	Wavefronts are planar, directional, and predictable.

Only in the far-field (i.e. when $d > d_f$) can we safely apply the Friis free space model.

Path Loss Consider Reference Distance

Reference distance d_0:
The Friis equation is not valid at $d = 0$, since P_r diverges. In practice, the signal near the transmitter is unstable due to near-field effects.

→ To avoid this, we define a small reference distance d_0, typically 1 m (indoor), 100 m (outdoor) or 1 km (large cellular system), where $P_r(d_0)$ is measured or computed.

→ Received power at any $d > d_0 > d_f$ can be estimated using:

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^2$$

→ Path Loss (in dB) in free space at $d > d_0 > d_f$:

$$PL(d)\,[\text{dB}] = PL(d_0)\,[\text{dB}] + 20\log\left(\frac{d}{d_0}\right)$$

General Path Loss Model

Log-distance Path Loss Model

Theoretical and measurement indicate that average path loss is a linear function of T-R separation distance d in logarithmic:

Path Loss Model

$$\overline{PL}(d)\ [\mathrm{dB}] = \overline{PL}(d_0)\ [\mathrm{dB}] + 10n\log\left(\frac{d}{d_0}\right)$$

- $\overline{PL}(d)$: Ensemble average of all possible path loss values for a given d
- d: T-R separation distance
- d_0: Reference distance
- n: Path loss exponent, indicates the rate of path loss increases as d increasing

For example, in free space, $n = 2$, and when obstructions are more, n will have a larger value.

General Path Loss Model

Log-distance Path Loss Model Consider Shadowing

Shadowing effects occur even at the same T-R separation d, where variations in obstacles cause the received signal strength to fluctuate depending on the location.
(e.g., with the TX at the center and RX located along the circumference)

Measurements show that path loss $PL(d)$ at a given location follows a log-normal distribution with mean $\overline{PL}(d)$:

Shadowing Path Loss Model

$$\begin{aligned} & PL(d)\,[\mathrm{dB}] \sim \mathcal{N}\left(\overline{PL}(d)\,[\mathrm{dB}],\, \sigma^2\right) \\ \Rightarrow\quad & PL(d)\,[\mathrm{dB}] = \overline{PL}(d)\,[\mathrm{dB}] + X_\sigma \\ & = PL(d_0)\,[\mathrm{dB}] + 10n\log_{10}\left(\frac{d}{d_0}\right) + X_\sigma, \quad X_\sigma \sim \mathcal{N}(0,\, \sigma^2) \end{aligned}$$

Path loss = average path loss + random shadowing (variation)

Computer Simulation and In Practice

In Computer Simulation:

- Parameters d_0, n, and σ are sufficient to statistically describe the path loss model.

In Practice:

- n and σ are obtained from measurement using MMSE of linear regression.
- $\overline{PL}(d_0)$ is based on a reasonable assumption.

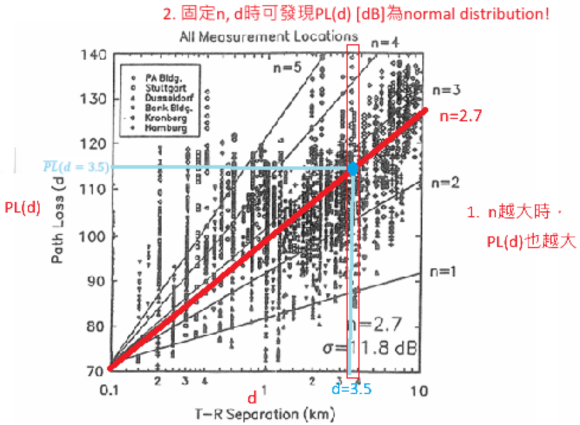


Figure: Scatter plot of measured data with many cities

Probability of Exceeding Signal Threshold

Because of log-normal shadowing, the received signal power $P_r(d)$ is a random variable with normal distribution in dB, centered at the average value $\overline{P_r(d)}$:

$$P_r(d)\,[\mathrm{dB}] \sim \mathcal{N}(\overline{P_r(d)}, \sigma^2)$$

To compute the likelihood that signal strength exceeds a threshold γ:

$$\Pr[P_r(d) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right)$$

- γ: minimum power level RX can decode
- $Q(\cdot)$: Q-function, i.e., tail probability of standard normal distribution
- σ: standard deviation of shadowing (in dB)

This helps predict how likely a user at a given distance receives sufficient signal.

Area coverage

For a circular cell of radius R, we define area coverage $U(\gamma)$ as the percentage of area where the received power exceeds threshold γ:

$$
\begin{aligned}
U(\gamma) &= \frac{1}{\pi R^2} \int_A \Pr[P_r(d) > \gamma]\, dA \\
&= \frac{1}{2}\left[1 - \mathrm{erf}(a) + \exp\left(\frac{1-2ab}{b^2}\right)\left(1 - \mathrm{erf}\left(\frac{1-ab}{b}\right)\right)\right] \\
&= \frac{1}{2}\left[1 + \exp\left(\frac{1}{b^2}\right)\left(1 - \mathrm{erf}\left(\frac{1}{b}\right)\right)\right] \quad \text{(when } a = 0\text{)}
\end{aligned}
$$

where

$$
a = \frac{\gamma - P_t + PL(d_0) + 10n\log(R/d_0)}{\sigma\sqrt{2}}, \quad b = \frac{10n\log e}{\sigma\sqrt{2}}
$$

Boundary coverage $\Pr[P_r(R)]$ refers to the probability (ratio) that, at the outer edge of the cicle (i.e., at d=R), the received power exceeds the threshold γ . When $\Pr[P_r(R)] = \gamma$, $a = 0$.

Example

Given measurements:

Distance from TX	Received Power p_i
100 m	0 dBm
200 m	-20 dBm
1000 m	-35 dBm
3000 m	-70 dBm

Note: $P[dBm] = 10\log(P[W]/1mW) = 10\log(P[W]) + 30$.

Assumption:

Assume the log-distance power received model (see appendix) $\hat{P}(d) = P(d_0) - 10n\log_{10}\left(\frac{d}{d_0}\right)$

Let reference distance $d_0 = 100$ m ,then $P(d_0) = 0$ dBm. Then:

Distance from TX	Predicted Power $\hat{p}_i$ (dBm)
100 m	0
200 m	$-3n$
1000 m	$-10n$
3000 m	$-14.77n$

Example: Compute Path loss exponent n

Using MMSE (minimum mean square error) to estimate path loss.

We define the squared error cost function (or just error function):

$$J(n) = (0-0)^2 + (-20+3n)^2 + (-35+10n)^2 + (-70+14.77n)^2$$

$$J(n) = 6525 - 2887.8n + 327.153n^2$$

Take derivative and solve:

$$\frac{dJ(n)}{dn} = 654.306n - 2887.8 = 0 \Rightarrow n = 4.4$$

Example: Compute Standard Deviation σ

To measure the error between theoretical and actual values, we use:

$$\sigma^2 = \frac{1}{k}\sum_{i=1}^{k}(p_i - \hat{p}_i)^2 = \frac{J}{k}$$

At $n = 4.4$:

$$J(4.4) = (0-0)^2 + (-20+13.2)^2 + (-35+44)^2 + (-70+64.988)^2 = 152.36$$

$$\sigma^2 = \frac{J}{4} = 38.09 \quad \Rightarrow \quad \sigma = \sqrt{38.09} = 6.17 \text{ dB}$$

Note: This is a biased estimate due to the small number of samples. (For an unbiased estimate, divide by $k - 1 = 3$ *instead of 4.)*

Example: Predict Received Power at 2 km

Using the log-distance path loss model:

$$\hat{P}(d) = P(d_0) - 10n\log\left(\frac{d}{d_0}\right)$$

$$\hat{P}(2000) = 0 - 10(4.4)\log\left(\frac{2000}{100}\right) = -57.24 \text{ dB}$$

Consider shadowing:
Simply adding a Gaussian random variable with zero mean and $\sigma = 6.17$ dB:

$$\hat{P}(2000) \text{ [dB]} \sim \mathcal{N}\left(-57.24,\ (6.17)^2\right)$$

Example: Estimate Area Coverage

Probability of $P(d) > -60$ dBm at 2 km

$$\Pr[P(d) > -60] = Q\left(\frac{-60 + 57.24}{6.17}\right) = Q(-0.45)$$

From Q-table:

$$Q(-0.45) = 1 - Q(0.45) \approx 1 - 0.326 = 0.674$$

If given that the boundary coverage $\Pr[P_r(R) > \gamma] = 67.4\%$
Then, area coverage $U(\gamma)$ is estimated using:

$$U(\gamma) = \frac{1}{2}\left[1 - \text{erf}(a) + \exp\left(\frac{1-2ab}{b^2}\right)\left(1 - \text{erf}\left(\frac{1-ab}{b}\right)\right)\right]$$

When $\Pr[P_r(R) > \gamma] = 67.4\%$, then area coverage $U(\gamma) \approx 88\%$
Insight: Users closer to the transmitter generally have stronger signals, so the average area coverage is greater than the boundary coverage.

Appendix: Equivalent Model

From p.11, we used the received power model:

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^n = P_r(d_0)\left(\frac{d}{d_0}\right)^{-n}$$

$$\Rightarrow P_r(d)\,[\mathrm{dB}] = P_r(d_0)\,[\mathrm{dB}] - 10n\log\left(\frac{d}{d_0}\right)$$

Equivalently, path loss is defined as the difference between transmitted and received power:

$$PL(d) = P_t - P_r(d) = P_t - P_r(d_0)\left(\frac{d}{d_0}\right)^{-n}$$

$$\Rightarrow PL(d)\,[\mathrm{dB}] = 10\log\left(P_t + P_r(d_0)\right) + 10n\log\left(\frac{d}{d_0}\right)$$

$$PL(d)\,[\mathrm{dB}] = PL(d_0)\,[\mathrm{dB}] + 10n\log\left(\frac{d}{d_0}\right)$$

The End